STUDY REGARDING PROPAGATION AND ATTENUATION OF A SHOCKWAVE AT THE IMPACT WITH A DURALUMINUM TARGET

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ABSTRACT

This paper presents the analytical study of shock waves resulting from the impact of a cylindrical projectile on a static target. In this analytical study, the materials are considered under hydrodynamic treatment (without elastic limit). The analytical study is complemented by numerical simulations using the ABAQUS code. Firstly, the corresponding configuration for various impedance ratios of the projectile and target material has been analyzed, subsequently the unit ratio material Al2024 (duralumin) being chosen. Thus the case where a projectile hits a target of the same material, namely duralumin is analysed. Shock wave propagation equations, reflection and attenuation of the shock wave in the projectile and target were determined. It is of interest to determine the distance traveled by the shock wave before attenuation, both in the target and the projectile. We noticed that the experimental results obtained with Taylor’s cannon are consistent with those obtained from simulations with the ABAQUS code.

KEYWORDS: shock, wave, propagation, attenuation

1. Introduction

A wave is a thermodynamic disturbance propagation (P, ρ, E ...) in a gaseous medium (a sound in the air), liquid (waves created by the fall of a body in water) or solid (earthquakes). When the wave carries a weak perturbation in relation to ambient pressure it qualifies as a sound wave.

A shock wave is the propagation of a discontinuity of characteristic quantities of a thermodynamic and mechanical condition of a medium. This discontinuity is a consequence of nonlinear effects that cause rigidity of sound waves and can generate intense mechanical effects.

A shockwave is a thin transitive area propagating with supersonic speed in which there is a sharp increase of density, pressure and speeds of substance. A mechanical or physical shock is a sudden acceleration or
deceleration caused, for example, by impact, drop, kick, earthquake, or explosion. Shock is a transient physical excitation. The energy of a shock wave dissipates relatively quickly with distance. Also, the accompanying expansion wave approaches and eventually merges with the shock wave, partially canceling it out. When a shock wave passes through matter, the total energy is preserved but the energy which can be extracted as work decreases and entropy increases.

An impact is a high force or shock applied over a short time period when two or more bodies collide. The effect depends critically on the relative velocity of the bodies to one another.

The way in which the kinetic energy is distributed through the section is also important in determining its response. Projectiles apply a Hertzian contact stress [1] (refers to the localized stresses that develop as two curved surfaces come in contact and deform slightly under the imposed loads) at the point of impact to a solid body, with compression stresses under the point, but with bending loads a short distance away. Since most materials are weaker in tension than compression, this is the zone where cracks tend to form and grow.

2. General Notions

In a solid medium, even in the absence of external forces, the constituent atoms are in strong interactions.

To compress a solid must therefore acted against repulsive forces that grow rapidly when the distances between the component particles decrease.

The shock compression process being adiabatic and irreversible, it is accompanied by an increase in temperature, the increase being more important as the pressure gets higher. It results therefore a thermal component of the pressure associated with the vibrations of atoms (and possibly the electrons), component that overlaps the elastic component.

Elastic pressure is dominant up to several hundred kbars, the medium’s compression ratio remaining small. The thermal component of the pressure, which increases with the amplitude of the shock, becomes equal to the elastic component of the pressure, reaching a few Mbars. Thus, the increase of internal energy of a dense medium under shock results from the elastic potential energy increase on the one hand and on the other hand the thermal energy, in proportions which depend on the levels of pressure reached.

Since the time of applying pressure is very short compared to the time in which the target moves and deforms, the inertial forces have a decisive role in the formation of the shock wave.

A shock separates the material in which it is propagating into a part through which it has passed as shown in the figure below.

**Fig. no. 1 Shock propagation**

where \( P \) – pressure, \( t \) – time, \( \rho \) – density, \( E \) – energy, \( u \) – particle velocity, \( D \) – shock velocity.
The particle velocity $u$ refers to the velocity that a given element in the material acquires, as a result of the shock wave passing over the element. The shock velocity is the velocity with which the disturbance moves through the body. The shock velocity is always greater than the particle velocity.

A wave is the propagation in a dense medium of a perturbation or discontinuity [2]. This disturbance modifies the physical characteristics of the medium and propagates a sudden change of pressure that can be increasing in the case of compression waves or decreasing in the case of waves of relaxation.

The wave is decomposed into elements of infinitely small amplitude which follow and spread a continuous and infinitesimal evolution of physical characteristics of the medium. These elements are called low-amplitude sound waves.

3. Conservation Equations

Thermodynamic variables or state variables which intervene in shock analytical relations are: pressure $p$, density $\rho$ (or specific volume $V$) and specific energy (energy per unit mass of matter). Specific density is also called absolute density.

State variables are: in front of the shock, medium’s undisturbed state, ($P_0$, $\rho_0$, $E_0$), and behind the shock, the disturbed state of the medium ($P_1$, $\rho_1$, $E_1$).

The shock wave is characterized by two kinematic variables:
– relative velocity $D$ to initial medium;
– velocity jump which the shock wave propagates in the medium.

The fundamental principles of mechanics are applied to the shock wave transformation: conservation of mass, momentum (quantity of motion) and energy. Thus of the three state variables and the two kinematic variables, it can be obtained three relationships that constitute as the analytical relations of the shock wave, also called HUGONIOT – RANKINE relations [3].

To simplify the mathematical relations, it is assumed that the medium is initially at rest [4], [5] [6].

These velocities are given in relation with an absolute reference.

a) Conservation of mass

$$\rho_0 \cdot D = \rho(D - u) \quad (1)$$

b) Conservation of momentum

The conservation of momentum requires that the difference in momentum be equal to the impulse per unit cross-sectional area:

$$\rho(D - u_0)(u_1 - u_0) = P - P_0 \quad (2)$$

If $u_0 = 0$

$$\rho_0 \cdot D \cdot u_1 = P - P_1 \quad (3)$$

where $\rho_0 \cdot D$ is often known as the shock impedance.

c) Conservation of energy

The conservation of energy is obtained by setting up an equation in which the work done by $P$ minus and the work done by $P_0$ is equal to the difference in the total energy (kinetic and internal) between the two sides of the front. There is no heat exchange with the external environment, the transformation being adiabatic.

$$E - E_0 = \frac{1}{2}(P + P_0)(V_0 - V) \quad (4)$$

where $V_0$, $V$ is the specific volume.

Hence, an additional equation is needed if one wants to determine all parameters as a function of one of them, expressed as the relationship between shock and particle velocities and it has to be experimentally determined:

$$D = c_0 + s \cdot u \quad (6)$$

The equation above is often known as the equation of state of a material, where $c_0$ is the sound velocity in the material at zero pressure, and $s$ is an empirical parameter.

If however one has more variables than equations, the equation of Mie-Grüneisen (a Mie-Grüneisen equation of state is linear in energy) can be used:
The Rayleigh line is a straight line connecting points corresponding to the initial and final states on a graph of pressure versus specific volume for a substance subjected to a shock wave.

One can observe that the shock polar slope $\rho_0 \cdot D$ is the shock impedance $Z$. One can frequently work with domains in which the pressures correspond to small particle velocities compared with the shock velocities: $u << D$. In the case of analytic studies, for simplifying the calculus, one can make the $Z = \rho_0 \cdot C_0$ hypothesis. Hence, the shock impedance is reduced to the acoustic impedance, the shock polar becomes a line.

5. Taylor’s Canon

The gas gun is an apparatus for physics experiments, a highly specialized gun designed to generate calibrated impacts. It is usually used to study impact phenomena on structures.

The scheme of the installation is shown in the figure below:
Where:
1 – Gas bottle
2 – Gas tank
3 – Electro valve
4 – Cannon’s tube
5 – Protection box
6, 7 – Pressure gauge
8 – Tap

5.1. Characteristics
- Length: 2000 mm
- Interior diameter: 50 mm
- Gas tank of 6L standardized to 20 bars.
- Possibility of changing the gas tank in order to use different gas (nitrogen, helium) with a higher pressure available but less than 80 bars.
- Height of gas evacuation tube: 210 mm
- 4 laser lens (2 transmitters and 2 receivers) for measuring the projectile’s at the exit from the cannon.

The Taylor’s cannon test consists of a projectile’s (cylinder) propulsion towards a target which is rigid and/or symmetric to the projectile.

To simulate the shock wave that Taylor’s cannon can generate we used the program ABAQUS version 6.8-3. It has been studied with ABAQUS the way the shock wave propagates in the material.

6. Under Shock State Determination
6.1. Hypotheses
- We consider that the tensile stresses experienced by the materials never exceed the rupture limit.
- We will consider the impact as perfectly plane.
- The problem will be treated as 1D (unidimensional).
- We will consider the shock polar lines.

6.2. Problem
A duralumin projectile (Al 2024 or Al 2017A) of 2 mm thickness is accelerated to an initial speed stabilized around $V_0 = 100$ m/s and impacts a target material, duralumin also, initially at rest.

6.3. Initial data
The data in the table below is used to find out the shock wave parameters (velocity, pressure, etc).

<table>
<thead>
<tr>
<th>Materials properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>Duralumin</td>
</tr>
<tr>
<td>Steel 304</td>
</tr>
<tr>
<td>Copper</td>
</tr>
</tbody>
</table>
where $\rho_0$ is the initial density, $C_0$ is the sound hydrodynamic velocity at zero pressure, $s$ is the slope of the $D_0-u$ variation ($C_0$, $s$ – material characteristic constants), and $Y_0$ is the Young’s modulus.

To determine the state under shock we consider the shock polars of the materials in the (P-u) plane. In our case the projectile and the target are both made of duralumin.

The equations used are:

$$
\begin{align*}
\rho C_0 &= \rho_0 C_0 (1 + s u) + \rho_0 \cdot s \cdot u^2 \\
C_0 &= \rho_0 C_0 (1 + s u) + \rho_0 \cdot s \cdot (u_0 - u)^2
\end{align*}
$$

(9)

Because we have similar materials for the projectile and target also, the shock impedance is the same, so there are no reflexions and one can find the pressure and the particle velocity $u$ with the help of the shock polar for the projectile and the shock polar’s symmetric for the target (polar reverse).

One can also use the formula:

$$
\rho C_0 = \frac{\rho_0 C_0}{2}
$$

(10)

The results are:

$P_1 = 0.751239825 \text{ GPa}$

$u_1 = 50 \text{ m/s}$

The values $P_1$ and $u_1$ correspond to state 1, as in the figure below, assuming that $P_0$ and $u_0$ are equal to zero.
Having the initial values from (Table no. 1) one can determine the values for the first state of propagation in the projectile and target.

\[
D_{01abs} = -D_{01} + V_0 = -[c_0 + s(u_1 - V_0)] + V_0
\] 

(11)

For the mass conservation in the projectile we use the formula expressed in the absolute reference:

\[
\rho_0(D_{01abs} - V_0) = \rho_1(V_{01abs} - u_1)
\] 

(12)

Hence, the equations for state 1 are:

\[
u_1 = \frac{V_0}{2}
\] 

(13)

\[
P_1 = \rho_0 \cdot c_0 \cdot (V_0 - u_1) + \rho_0 \cdot s \cdot (V_0 - u_1)^2
\] 

(14)

\[
D_{01abs} = -[c_0 + s(u_1 - V_0)] + V_0
\] 

(15)

\[
D_{01} = c_0 \cdot s \cdot u_1
\] 

(16)

\[
\rho_1 = \rho_0 \cdot \frac{D_{01} - V_0}{D_{01abs} - V_0}
\] 

(17)

\[
c_1 = \frac{\rho_1}{\rho_0} (c_0 + 2 \cdot s \cdot u_1)
\] 

(18)

And the values are:

<table>
<thead>
<tr>
<th>Quantities numeric values in the first state of propagation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_0 ) (m/s)</td>
</tr>
<tr>
<td>( u_1 ) (m/s)</td>
</tr>
<tr>
<td>( D_{01abs} ) (m/s)</td>
</tr>
<tr>
<td>( D_{01} ) (m/s)</td>
</tr>
<tr>
<td>( P_1 ) (GPa)</td>
</tr>
<tr>
<td>( \rho_1 ) (kg/m3)</td>
</tr>
<tr>
<td>( c_1 ) (m/s)</td>
</tr>
</tbody>
</table>

![Space – time diagram of the duralumin-duralumin (projectile – target) impact](image-url)
7. Shock Wave Attenuation

Shock attenuation occurs when a shock is overtaken by a smooth decompression wave. At impact, shock waves form at the interface and propagate forward into the target and backward into the projectile plate. When the receding shock encounters the unrestrained back surface of the projectile plate, a centered simple decompression wave forms and propagates forward. Since the sound speed in the compressed material behind the advancing shock exceeds the shock velocity, the smooth wave will eventually overtake the shock. Because the overtaking wave is one of decompression, we expect that the interaction will cause a decrease in shock strength and velocity.

Because of the varying shock strength, the flow behind the shock will not be isentropic. This complication is avoided in the weak-shock approximation that we shall adopt.

One can study the attenuation of one dimensional shock wave produced by the flying plate impact in solids. The study is based on the assumption that the behavior of the solids is hydrodynamic in the range of pressure considered and the attenuation of shock pressure occurs due to its interaction with the rarefaction wave.

The attenuation experiments can be simulated by the use of a computer code, in case in which the rigidity is neglected.

Having the values for state 1 we can find out the distance \( R \) for the attenuation in the duralumin using the equation from [9].

\[
R = \frac{2c_s^2 - \rho_0}{\rho_0 c_s^2 + D_{D_P}}
\]  

(19)

where \( d = 0.001 \) m, projectile’s tickness.

It results \( R = 0.163282511 \) m.

If we want to calculate the distance (the point where the rarefaction wave catches the shock front) using the values given by the shock polars, (Table no. 1) and (Table no. 2) we use the equation:

\[
t = ax + b,
\]

where \( t = \) time, \( x = \) distance.

Hence, with the system formed we can find out the coordinates of point \( A \) and \( B \):

\[
\begin{align*}
\frac{t_B}{D_{D_P}} &= \frac{\rho_0}{c_s^2 + u_s^2} \\
\frac{t_A}{D_{D_P}} &= \frac{\rho_0}{c_s^2 + u_s^2} \\
\frac{x_A}{V_s - D_{D_P} u_s} &= d \\
\frac{x_B}{V_s - D_{D_P} u_s} &= \frac{t}{D_{D_P}}
\end{align*}
\]

where: \( B \) – the point of attenuation in the target, \( A \) – the point of attenuation in the projectile.

It results:

\[
\begin{align*}
t_B &= 3.00807E-05 \text{ s} \\
x_B &= 0.162282511 \text{ m}
\end{align*}
\]
The slightly difference between \( x_B \) and \( R \) is because we use the Eulerian coordinates, with the absolute reference \( D_0 \). 

With the help of ABAQUS we were able to determine how the attenuation takes place in the duralumin target and projectile and also how the shock wave propagates in both bodies.

To draw the P-x diagram the highest pressure values were taken beginning with the attenuation point.

\[
y = 910332x^3 - 51889x^2 + 870.51x - 3.5517
\]

**Fig. no. 8** P-x diagram of the pressure profile in the target

After adding a trendline we can observe that our profile pressure can be approximated as a degree 3 equation.

Because we normalized the profile 1 being maximum and 0 being minimum, we can observe that the point \( x = 0.5 \) can be taken as an inflection point \( \left( \frac{d^2 P(x)}{dx^2} = 0 \right) \).

So our equation can be written as:

\[
P(x) = ax^3 + bx^2 + cx + d
\]

where \( a, b, c, d \) depend on the impact velocity \( V_0 \).

The attenuation equation in duralumin is valid up to initial velocities of \(~250\text{m/s} \).

**Fig. no. 9** Profile of attenuation in duralumin with the help of ABAQUS
With the ABAQUS code we also studied how the shock wave propagates in the target and it can be observed that the transmitted wave is planar, having the characteristics: \( V_0 = 100 \text{ m/s}; \) projectile thickness – 0.002 m; target thickness – 0.1 m; axisymmetric – diameter of 0.04 m; finite element mesh size – 0.0001 m.

![Fig. no. 10 Shock wave propagation in duralumin target](image)

8. Conclusions

This paper contains an analysis on the properties and transmission of shock wave and a discussion of how it interacts after a flyer plate impact onto a duralumin target.

Based on the presented mathematical model it had been determined how the shock wave propagates through a material of duralumin. We have done several simulations in Abaqus, concerning the dimensions of the pieces, different methods (e.g. penalty, kinematics’ method, etc), finite elements sizes, different parameters (e.g. shear modulus).

We calculated how the shock wave propagates in the material, having the certitude that the results are correct because it was done from several points of view.

Finally, based on the realised studies and experiments we propose a quite approximate method to evaluate the attenuation and evolution of the shock wave precursor under the assumption that the entropy jump across the wave is negligible.

It is interesting to note that the approximate method described above is useful for calculating the attenuation in duralumin up to a velocity of \(~ 250\text{m/s},\) the value being superior to other velocities used in other known methods of determining the attenuation of shock wave.

We dealt with the symmetrical impact problem. It means that the driver plate and target plate are made of the same material and in the same thermodynamic state.

The attenuation, which occurs after the shock wave passes through the material, is more detailed, the simulations in ABAQUS, being an important part in the analysis. We observed that for a more precise assay a greater number of finite elements should be used.

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